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HEAT TRANSFER BETWEEN PARALLEL PLATES INCLUDING RADIATION AND RAREFACTION EFFECTS

RALPH GREIF* and D. R. WILLIS†
University of California, Berkeley, California

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Abstract—We consider the heat transfer through a gas contained between two parallel plates. We assume a gray gas with the radiative emission corresponding to local thermodynamic equilibrium. The conduction in the gas is treated by using a kinetic theory approach based on a Bhatnagar–Gross–Krook model. The radiative and kinetic transport equations are coupled through the local "temperature" which appears in both emission terms. The problem is formulated via a Milne–Eddington–Lees moment method. The optical thickness, the Knudsen number and the ratio of the continuum conduction to the continuum radiative heat transfer are parameters in the problem. Specific solutions are obtained for the case when the ratio of the temperature difference between the plates to a characteristic temperature is small. These solutions are compared with approximate results.

NOMENCLATURE

A,	approximation for intensity, equation (11);
a,	non-dimensional intensity perturbation, $[A/(\sigma T_0^4/\pi)] - 1$;
b,	constant, $32/15\pi$;
D,	distance between plates;
f,	velocity distribution function;
I,	intensity of radiation;
k,	Boltzmann constant;
k_c,	thermal conductivity coefficient;
K,	absorption coefficient;
m,	mass of molecule;
n,	number density;
Q,	heat flux;
q,	non-dimensional heat flux, $\beta Q/\sigma T_0^4$;
Pr,	Prandtl number;
R,	specific gas constant, k/m;
T,	temperature;
t,	non-dimensional temperature perturbation, $(T/T_0) - 1$;
y,	spatial coordinate.

Greek symbols

α,	parameter defined in equation (27);
β,	parameter defined in equation (27);
δ,	constant appearing in the collision frequency defined in equation (32);
ϵ,	$2\|T_h - T_c\|/(T_h + T_c)$;
μ,	direction cosine;
σ,	Stefan–Boltzmann constant;
ν,	non-dimensional density perturbation, $(n/n_0) - 1$;
ξ_y,	molecular velocity;
$d^3\xi$,	elemental volume;
τ,	optical depth, $\int K\,dy$;
$d\omega$,	elemental solid angle;
Ω,	parameter defined in equation (37).

Subscripts

c,	lower plate;
cond,	conduction;
h,	upper plate;
M,	Maxwellian;
0,	reference conditions;
rad,	radiation.

* Assistant Professor of Mechanical Engineering.
† Associate Professor of Aeronautical Sciences.

INTRODUCTION

THE TRANSFER of energy in an absorbing, emitting and conducting medium is of importance in many engineering problems. The complexity of the interaction between conduction and radiation has been discussed in several investigations; see, for example, the review articles by Viskanta and Grosh [1, 2] and Cess [3]. In all these studies the contribution from conduction has been considered from the point of view of a continuum using the Fourier law of heat conduction. In this study we shall examine the more general problem using a kinetic theory description and recover the Fourier law results as a limiting case.

ONE DIMENSIONAL EQUATIONS

We consider the transfer of energy by photons and molecules in the one-dimensional system bounded by two isothermal plane parallel plates. The plate at $y = D/2$ is maintained at a temperature T_h; at $y = -D/2$ the plate temperature is T_c.

The equation of radiative transfer for a gray, non-scattering medium is given by*

$$\frac{\mu}{K}\frac{\mathrm{d}I}{\mathrm{d}y} = -I + \frac{\sigma T^4}{\pi} \tag{1}$$

where we have used the source function corresponding to local thermodynamic equilibrium. The black body intensities leaving the two plates are specified by

$$\left.\begin{aligned} I(-D/2) &= \sigma T_c^4/\pi, \qquad \mu > 0 \\ I(+D/2) &= \sigma T_h^4/\pi, \qquad \mu < 0. \end{aligned}\right\} \tag{2}$$

The kinetic theory distribution function should be determined from the solution of the Boltzmann equation. We, however, use a model of the type proposed by Krook [4] to describe the collisions and obtain

$$\xi_y \frac{\partial f}{\partial y} = -\,\delta n[f - f_M(n, T)] \tag{3}$$

where f_M is the Maxwellian velocity distribution function given by

$$f_M(n, T) = n(m/2\pi kT)^{\frac{3}{2}} \exp\left[-m\xi^2/2kT\right]. \tag{4}$$

The distribution function satisfies the plate conditions

$$\left.\begin{aligned} f(-D/2) &= f_M(n_c, T_c), \qquad \xi_y > 0 \\ f(+D/2) &= f_M(n_h, T_h), \qquad \xi_y < 0 \end{aligned}\right\} \tag{5}$$

and by conservation of mass

$$n = \int f\,\mathrm{d}^3\xi. \tag{6}$$

The equation for the conservation of energy is given by

$$\frac{\mathrm{d}}{\mathrm{d}y}(Q_{\mathrm{cond}} + Q_{\mathrm{rad}}) = 0 \tag{7}$$

where

$$Q_{\mathrm{cond}} = \int \tfrac{1}{2}m\xi^2\xi_y f\,\mathrm{d}^3\xi \tag{8}$$

and

$$Q_{\mathrm{rad}} = \int I\mu\,\mathrm{d}\omega. \tag{9}$$

In the equation of radiative transfer and in the modified Boltzmann equation we have used a single "temperature" T. This should be a fair approximation for systems having small non-equilibrium effects.* A relation for the temperature may be obtained from the radiation, modified Boltzmann and energy equations:

$$(\tfrac{3}{2})\,\delta n^2kT + 4K\sigma T^4 = \delta n \int \tfrac{1}{2}m\xi^2 f\,\mathrm{d}^3\xi + \int IK\,\mathrm{d}\omega. \tag{10}$$

An accurate numerical solution to the above system of equations subject to the given boundary conditions appears to be quite difficult to obtain and is perhaps unnecessary in view of the successful application and accuracy of moment methods in both radiating and rarefied gas flows. We, therefore, seek approximate solutions to the equations by assuming the following form for I and f:

* The direction cosine, μ, is defined as $\cos\theta$ where θ is the angle measured from the positive y direction.

* See, for example, Kulander [5] for a discussion of non-equilibrium effects.

$$\left.\begin{aligned} I^+(\mu, y) &= A^+(y), & \mu > 0 \\ I^-(\mu, y) &= A^-(y), & \mu < 0 \\ f^+(\xi, y) &= f_M(n_1, T_1), & \xi_y > 0 \\ f^-(\mu, y) &= f_M(n_2, T_2), & \xi_y < 0. \end{aligned}\right\} \quad (11)$$

These expressions are suggested by the Milne–Eddington [6] and Lees [7] type of analysis. The "two-sided" nature of I and f, which is a feature of the optically thin and free molecular solutions, is retained in these approximations. The variables, I^+, I^-, n_1, n_2, T_1, and T_2, are obtained from moment equations as in the Milne–Eddington and Lees methods (MELEES). These equations, together with the boundary conditions, are

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}\tau}(A^+ - A^-) = -(A^+ + A^-) + \frac{2\sigma T^4}{\pi} \quad (12)$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau}(A^+ + A^-) = -\tfrac{3}{2}(A^+ - A^-) \quad (13)$$

$$n_1 T_1^{\frac{1}{2}} - n_2 T_2^{\frac{1}{2}} = 0 \quad (14)$$

$$n_1 T_1 + n_2 T_2 = \text{constant} \quad (15)$$

$$\frac{\mathrm{d}Q_{\text{cond}}}{\mathrm{d}\tau} = \frac{\mathrm{d}}{\mathrm{d}\tau}\left\{\frac{m}{2\pi^{\frac{1}{2}}}\left[n_1(2RT_1)^{\frac{3}{2}} - n_2(2RT_2)^{\frac{3}{2}}\right]\right\} \quad (16)$$

$$= -\frac{3\delta nk}{4K}\left[n_1T_1 + n_2T_2 - 2nT\right] \quad (17)$$

$$= -\frac{\mathrm{d}Q_{\text{rad}}}{\mathrm{d}\tau} = \pi\left[2(A^+ + A^-) - \frac{4\sigma T^4}{\pi}\right] \quad (18)$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\left[\tfrac{5}{4}m(n_1R^2T^2 + n_2R^{\frac{1}{2}}T_2^2)\right]$$
$$= -\frac{\delta n}{K}\left\{\frac{m}{2\pi^{\frac{1}{2}}}\left[n_1(2RT_1)^{\frac{3}{2}} - n_2(2RT_2)^{\frac{3}{2}}\right]\right\} \quad (19)$$

where

$$\left.\begin{aligned} \mathrm{d}\tau &= K\,\mathrm{d}y \\ Q_{\text{rad}} &= \pi(A^+ - A^-) \\ T_1(-D/2) &= T_c \\ T_2(+D/2) &= T_h \\ A^+(-D/2) &= \sigma T_c^4/\pi \\ A^-(+D/2) &= \sigma T_h^4/\pi. \end{aligned}\right\} \quad (20)$$

This set of equations, while significantly simpler than the exact equations, is still quite difficult to solve.*

THE LINEARIZED PROBLEM

When $|T_h - T_c|/(T_h + T_c)$ is small we can linearize the equations and boundary conditions. This introduces important simplifications yet still retains the basic features of the problem. Certainly, the linear results should approximately reproduce the behavior of the non-linear system for the case of small temperature differences. Indeed, for the case when conduction dominates it has been shown [8] that the linearized moment method solution can be applied in highly non-linear problems ($T_h/T_c \simeq 10$) to give the heat transfer, according to the moment solution, within an error of 10 per cent.

We now define perturbation quantities t, v and a according to

$$\left.\begin{aligned} T &= T_0(1 + t) \\ n &= n_0(1 + v) \\ A &= \frac{\sigma T_0^4}{\pi}(1 + a) \end{aligned}\right\} \quad (21)$$

and obtain the following set of equations

$$\frac{\mathrm{d}}{\mathrm{d}\eta}(a^+ - a^-) = \tau_0[16t - 2(a^+ + a^-)] \quad (22)$$

$$\frac{\mathrm{d}}{\mathrm{d}\eta}(a^+ + a^-) = -\frac{3\tau_0}{2}(a^+ - a^-) \quad (23)$$

$$\frac{\mathrm{d}}{\mathrm{d}\eta}(t_1 - t_2) = -\alpha(t_1 + t_2 - 2t) \quad (24)$$

$$\frac{\mathrm{d}}{\mathrm{d}\eta}(t_1 + t_2) = -\alpha b(t_1 - t_2) \quad (25)$$

$$q = t_1 - t_2 + \beta(a^+ - a^-) \quad (26)$$

* We are currently attempting to solve these non-linear equations numerically.

where

$$\left.\begin{aligned} &T_0 = (T_h + T_c)/2, \quad \epsilon = (T_h - T_c)/T_0 \\ &\eta = y/d = \tau/\tau_0, \quad \alpha = 3\delta n_0 D\pi^{\frac{1}{2}}/4(2RT_0)^{\frac{1}{2}} \\ &\beta = \sigma T_0^4 \pi^{\frac{1}{2}}/\rho_0 RT_0(2RT_0)^{\frac{1}{2}}, \quad b = 32/15\pi \\ &Q = (2RT_0)^{\frac{1}{2}}(\rho_0 RT_0)q/\pi^{\frac{1}{2}} = \sigma T_0^4 q/\beta \end{aligned}\right\} \quad (27)$$

and the boundary conditions are

$$\left.\begin{aligned} t_1(-1/2) &= -\varepsilon/2 \\ t_2(+1/2) &= \varepsilon/2 \\ a^+(-1/2) &= -2\varepsilon \\ a^-(+1/2) &= 2\varepsilon. \end{aligned}\right\} \quad (28)$$

The preceding system of linear equations may be solved directly and we obtain

$$\frac{-Q_{\text{MELEES}}}{\sigma T_0^4 \epsilon} = \frac{-q_{\text{MELEES}}}{\beta\epsilon} = \frac{(A/\beta) + 4B}{Af(1 + \alpha b/2) + Be\beta(1 + 3\tau_0/4)} \quad (29)$$

where

$$\left.\begin{aligned} A &= (3\tau_0/2l)\tanh(l/2) + 1 \\ B &= (\alpha b/l)\tanh(l/2) + 1 \\ e &= 8b\alpha/(3\tau_0 + 8b\beta\alpha) \\ f &= 3\tau_0/(3\tau_0 + 8b\beta\alpha) \\ l^2 &= (3\tau_0^2\alpha + 8b\beta\tau_0\alpha^2)/(\alpha + 8\tau_0\beta) \end{aligned}\right\} \quad (30)$$

DISCUSSION OF RESULTS AND CONCLUSIONS

Before discussing the general problem and results we first consider the important continuum or no-slip limit. In this limit the value of the ratio of the particle mean free path, λ, to the characteristic length, D, is zero, that is, zero Knudsen number. From equation (27) we have

$$\alpha = \frac{3}{4}\frac{\delta n\pi^{\frac{1}{2}}D}{(2RT_0)^{\frac{1}{2}}} \quad (31)$$

and choosing δ to match the continuum (Chapman–Enskog) thermal conductivity for a monatomic gas we obtain

$$\delta = Pr\,.\,kT_0/\mu(T_0). \quad (32)$$

Using $Pr = \frac{2}{3}$ and

$$\mu(T_0) = \frac{nm}{2}(8kT_0/\pi m)^{\frac{1}{2}}\lambda \quad (33)$$

yields

$$\alpha = \pi D/4\lambda \quad (34)$$

so that the no-slip limit corresponds to an infinite value for α. The corresponding value for the thermal conductivity is given by

$$k_c = 2D\sigma T_0^3/\alpha\beta b. \quad (35)$$

The non-dimensional heat flux in the no-slip limit is given by

$$-\frac{Q_{\text{no slip}}}{\sigma T_0^4\epsilon} = -\frac{q_{\text{no slip}}}{\beta\epsilon} = \frac{[8 + (4\Omega/3\tau_0)\{L\coth(L/2) + 3\tau_0/2\}](4 + \Omega)}{8(1 + 3\tau_0/4) + \Omega[L\coth(L/2) + 3\tau_0/2]} \quad (36)$$

where

$$\Omega = \frac{3k_cK}{4\sigma T_0^3} = \left(\frac{3}{2b}\right)\cdot\left(\frac{\tau_0}{\alpha\beta}\right) \quad (37)$$

and

$$L^2 = \tau_0^2(3 + 12/\Omega). \quad (38)$$

This result is of special importance because it demonstrates the parameters and their analytic dependence in the combined conduction and radiation problem. The parameters are the optical depth, τ_0, and the ratio denoted by Ω which is a measure of the importance of continuum conduction in comparison to optically thick radiation.*

Probstein [9] calculated the net continuum heat-transfer rate by neglecting the interaction between conduction and radiation and simply added the two "separate" heat fluxes in parallel.

* It should be noted that various combinations of Ω and τ_0 are present.

Good agreement with accurate numerical calculations was noted for a particular ratio of the plate temperatures. Also see Einstein [10] for additional calculations. Applying this procedure to our problem gives

$$-\frac{Q_{\text{no slip add.}}}{\sigma T_0^4 \epsilon} = -\frac{q_{\text{no slip add.}}}{\beta\epsilon} = 4\left[\frac{1}{1 + 3\tau_0/4} + \frac{\Omega}{3\tau_0}\right]. \quad (39)$$

In Table 1 the relative difference between the no slip and the no slip additive heat fluxes as

Table 1. Relative difference between no slip and additive heat fluxes

$1 - (Q_{\text{no slip add.}}/Q_{\text{no slip}})$.

Ω	$\tau_0 = 0{\cdot}1$	$\tau_0 = 1{\cdot}0$	$\tau_0 = 10{\cdot}0$
0·001	0·014	0·010	0·002
0·010	0·025	0·029	0·006
0·100	0·022	0·070	0·017
1·000	0·007	0·081	0·030
10·000	0·001	0·022	0·015
100·000	0·000	0·003	0·002

calculated from the expressions in equations (39) and (36) is presented. We see from this table that for the conditions considered the additive approximation gives results which are less than those obtained from the MELEES method. However, the maximum difference between the results is 8·1 per cent and occurs for $\tau_0 = 1{\cdot}0$ and $\Omega = 1{\cdot}0$. Note that for the linearized problem this result is independent of the plate temperature. That the maximum difference occurs for moderate values of τ_0 and Ω is readily explained. For large values of the optical depth, τ_0, the diffusion or Rosseland approximation is valid and the independent addition procedure is correct. For small values of τ_0 there is little absorption and consequently little interaction so that the independent addition is a good approximation. When Ω is large, conduction dominates, and since equation (39) contains the correct continuum conduction term errors should be small. Similarly, when Ω is small and τ_0 is large in comparison to $\Omega^{\frac{1}{2}}$, radiation dominates and equation (39) contains the correct limiting terms. When Ω is small and τ_0 is small in comparison to $\Omega^{\frac{1}{2}}$, equation (39) again contains the correct limiting terms.*

We return to the more general problem and note that in addition to the parameters τ_0 and Ω (and combinations of them) occurring in the no slip analysis we must also consider the parameter α, which is inversely proportional to the Knudsen number. The heat flux as determined from equation (29) is presented in

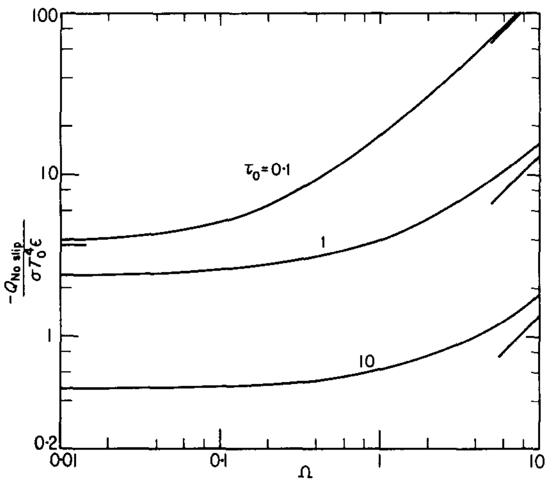

FIG. 1. No slip heat flux vs. Ω for $\tau_0 = 0{\cdot}1$, 1·0 and 10·0.

Figs. 2–4 for values of the optical depth, τ_0, of 0·1, 1·0, and 10·0, respectively. For large values of α, small particle mean free path, the heat flux correctly merges with the no slip curves of Fig. 1. For small values of α, large particle mean free path, the heat flux remains essentially constant until conduction becomes important.

A simple approximate solution for the general problem can be obtained by neglecting the coupling between conduction and radiation and adding the two heat fluxes as was done for the continuum limit. The relation for the heat

* For this case, $\Omega \ll 1$ and $\tau_0 \ll \Omega^{\frac{1}{2}}$, radiation dominates only when $\Omega/\tau_0 \ll 1$.

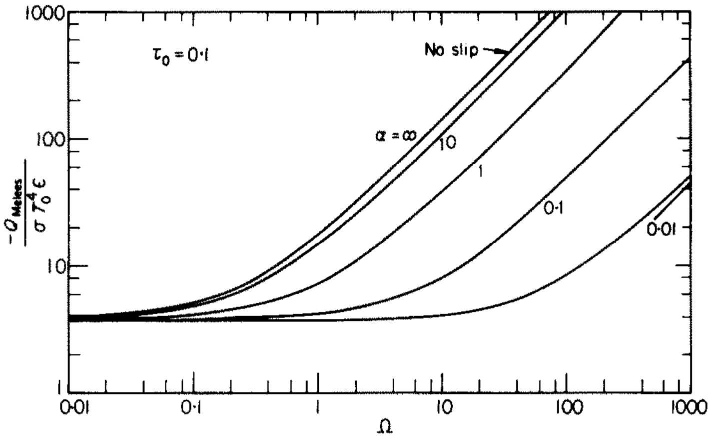

FIG. 2. Milne–Eddington–Lees heat flux vs. Ω for $\alpha = 0{\cdot}01$, $0{\cdot}1$, $1{\cdot}0$, $10{\cdot}0$ and ∞ for an optical depth of 0·1.

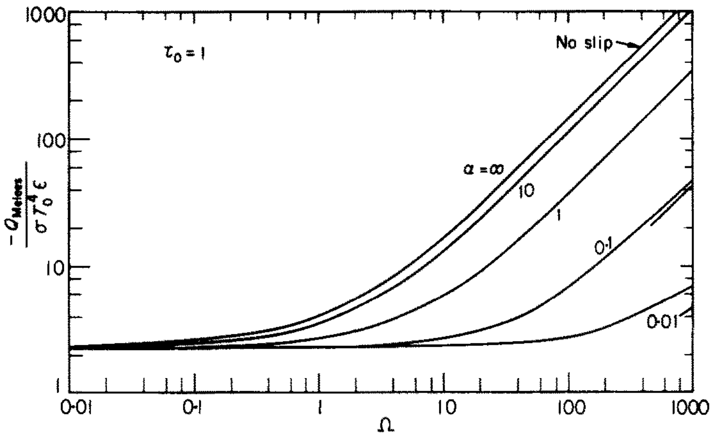

FIG. 3. Milne–Eddington–Lees heat flux vs. Ω for $\alpha = 0{\cdot}01$, $0{\cdot}1$, $1{\cdot}0$, $10{\cdot}0$ and ∞ for an optical depth of 1·0.

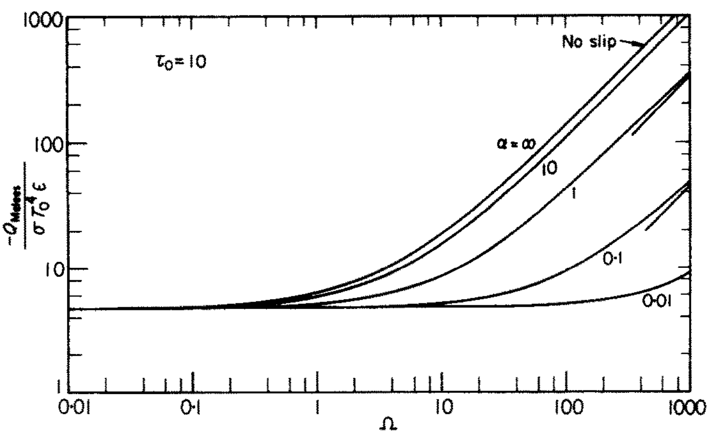

FIG. 4. Milne–Eddington–Lees heat flux vs. Ω for $\alpha = 0{\cdot}01$, $0{\cdot}1$, $1{\cdot}0$, $10{\cdot}0$ and ∞ for an optical depth of 10·0.

Table 2. Relative difference between Milne–Eddington–Lees and additive heat fluxes $(Q_{add.}/Q_{MELEES})$

α	Ω 0·001	0·01	0·1	1·0	10·0	100·0	1000·0
			(a) $\tau_0 = 0{\cdot}1$				
0·001	0·000	0·000	0·000	0·000	0·000	0·000	0·000
0·010	0·000	0·000	0·000	0·000	0·000	0·000	0·000
0·100	0·000	0·000	0·000	0·000	0·000	0·000	0·000
1·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000
10·000	0·003	0·011	0·013	0·005	0·000	0·000	0·000
100·000	0·011	0·023	0·021	0·006	0·000	0·000	0·000
1000·000	0·013	0·024	0·022	0·007	0·000	0·000	0·000
			(b) $\tau_0 = 1{\cdot}0$				
0·001	0·000	0·000	0·000	0·000	0·000	0·000	0·000
0·010	0·000	0·000	0·000	0·000	0·000	0·000	0·000
0·100	0·000	0·000	0·000	0·000	0·000	0·002	0·000
1·000	0·000	0·000	0·000	0·000	0·002	0·001	0·000
10·000	0·000	0·001	0·010	0·026	0·010	0·001	0·000
100·000	0·003	0·017	0·057	0·073	0·021	0·002	0·000
1000·000	0·009	0·028	0·068	0·080	0·022	0·003	0·000
			(c) $\tau_0 = 10{\cdot}0$				
0·001	0·000	0·000	0·000	0·000	0·000	0·000	0·000
0·010	0·000	0·000	0·000	0·000	0·000	0·000	0·001
0·100	0·000	0·000	0·000	0·000	0·001	0·006	0·011
1·000	0·000	0·000	0·001	0·006	0·032	0·048	0·037
10·000	0·000	0·000	0·001	0·005	0·009	0·002	0·000
100·000	0·000	0·000	0·003	0·013	0·008	0·001	0·000
1000·000	0·001	0·004	0·014	0·028	0·014	0·002	0·000

flux is easily obtained and is given by

$$-\frac{Q_{add.}}{\sigma T_0^4 \epsilon} = -\frac{q_{add.}}{\beta\epsilon} = 4\left[\frac{1}{(1 + 3\tau_0/4)} + \frac{b\alpha\Omega}{6\tau_0}\frac{1}{(1 + \alpha b/2)}\right]. \quad (40)$$

In Table 2 the relative difference between the Milne–Eddington–Lees and the additive heat fluxes as calculated from the expressions in equations (29) and (40) is presented. For the conditions calculated, the maximum difference is 8 per cent and occurs at $\tau_0 = 1{\cdot}0$, $\Omega = 1{\cdot}0$, and $\alpha = 1000{\cdot}0$. Indeed, for this value of α the no slip formulation may be used.

Therefore, in general and in particular for no slip conditions, the simple addition of two separate heat fluxes gives good agreement with the Milne–Eddington–Lees formulation for small temperature differences. However, the accuracy of this approximation when used in more complex situations is still uncertain.

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Résumé—Nous considerons le transport de chaleur à travers un gaz contenu entre deux plaques parallèles. Nous supposons que la gaz est gris avec une émission de rayonnement correspondant à l'équilibre thermodynamique local. On traite la conduction dans le gaz en employant une théorie cinétique bassée sur un modèle de Bhatnagar–Gross–Krook. Les équations de transport cinétique et par rayonnement sont couplées par le moyen de la "température" locale qui apparaît dans les deux termes d'émission. Le problème est formulé au moyen d'une méthode des moments du type Milne–Eddington–Lees. L'épaisseur optique, le nombre de Knudsen et le rapport de la conduction en régime continu au transport de chaleur par rayonnement en régime continu sont des paramètres du problème. On obtient des solutions spécifiques dans le cas où le rapport de la différence de température entre les plaques à une température caractéristique est faible. Ces solutions sont comparées avec des résultats approchés.

Zusammenfassung—Es wird der Wärmedurchgang durch eine Gasschicht zwischen zwei parallelen Platten betrachtet. Dabei wird das Gas als grau mit einer Emission, entsprechend dem örtlichen thermodynamischen Gleichgewicht angenommen. Die Leitung im Gas wird durch eine kinetische Theorie auf Grund eines Bhatnagar–Gross–Krook-Modells berücksichtigt. Die Gleichungen für die Strahlung und den Transport kinetischer Energie sind für die örtliche "Temperatur" gekoppelt, die in beiden Ausdrücken erscheint. Das Problem wird formuliert mit Hilfe einer Milne–Eddington–Lees Momentenmethode. Die optische Dicke, die Knudsen–Zahl und das Verhältnis von kontinuierlicher Leitung zu kontinuierlicher Strahlung stellen Parameter des Problems dar. Spezielle Lösungen werden für den Fall erhalten, dass das Verhältnis von Temperaturdifferenz zwischen den Platten zu einer charakteristischen Temperatur klein ist. Diese Lösungen werden mit Näherungsergebnissen verglichen.

Аннотация—Рассмотрен теплообмен через газ, заключенный между двумя параллельными пластинами. Газ считается серым с излучением, соответствующим локальному термодинамическому равновесию. Теплопроводность в газе трактуется в приближении кинетической теории, основанном на модели Батнагара–Гросса–Крука. Уравнения лучистого теплообмена и кинетики связаны через локальную температуру, входящую в оба члена, характеризующие излучение. Задача формулируется методом моментов Мильна–Эддингтона–Лиса. Оптическая толщина, число Кнудсена и отношение теплообмена теплопроводностью к теплообмену излучением являются параметрами задачи. Получены решения для частного случая небольшого отношения разности температур пластин к характерной температуре. Эти решения сравниваются с приближенными результатами.